THE OVERHEAT INSTABILITY IN AN OPTICALLY DENSE PLASMA DISCHARGE

S. A. Triger

The energy fluctuation per unit volume is calculated for an optically dense plasma discharge when the mean free path of quanta l is less than the characteristic dimensions of the system, and the wavelength of oscillations in the direction of inhomogeneity is $\lambda_x < l$. It is shown that the energy fluctuation is completely determined by the temperature fluctuation.

The dispersion relation describing the plasma oscillations is similar in structure to the corresponding equation in an optically transparent discharge [1] and contains the overheat instability.

The problem of the equilibrium and stability of a heavy current discharge in a comparatively low-temperature dense plasma, when the Rosseland mean free path of light in the medium l is less than the characteristic dimension of the system, was treated in [2, 3]. In this case, it turned out to be possible to use the approximation of radiative thermal conductivity [4] to describe such a discharge in its steady state.

The condition $\lambda_{min} > l$ (λ_{min} is the minimum wavelength characterizing the oscillations) was a necessary requirement for using this approximation in describing oscillations of the plasma discharge. When these conditions are fulfilled, the discharge can be regarded as optically dense both for the equilibrium state as well as for oscillations. It was shown in papers [2, 3] that power instabilities develop in such a discharge, associated with the impossibility of compensating magnetic pressure by kinetic plasma pressure in the oscillations. For a simple cylindrical discharge, the increments of these instabilities are very large, $\gamma \approx v_s/a \approx 10^5$ sec^{-1} (where a is the characteristic dimension of the discharge and v_s is the velocity of sound). However, creation of a discharge with an inverse axial current enables the increments due to the geometrical factor to be reduced substantially. Moreover, the finite conductivity of the plasma leads to stabilization of the high constructive modes.

In this connection, it is of interest to investigate whether an overheat instability could develop in an optically dense discharge, since it could play a fundamental part in these conditions. Such an instability was found in a transparent plasma discharge [1], and its increment attained magnitudes $\gamma \gtrsim 10^6$ sec^{-1}. The reason for the development of an overheat instability is the smallness of the radiative energy flux, which is unable to compensate for the temperature fluctuations.

The treatment given in [2, 3] showed that for conditions when $l < a$ and $l < \lambda_{min}$ there is no overheat instability, since the temperature oscillations relax rapidly because of the large radiative thermal conductivity. Thus, if the overheat instability occurs in an optically dense discharge for the equilibrium conditions obtained in [2], it should develop for wavelengths $\lambda_{min} < l$. In this case radiation does not only take place from the surface of the perturbed region of plasma. Quanta originating inside this region leave it freely (although they do not pass beyond the limits of the discharge itself) and so it can be said that the radiation has a volume nature in the perturbations and, as we shall see, cannot damp the temperature fluctuations.

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Teknicheskoi Fiziki, Vol. 11, No. 2, pp. 39–41, March-April, 1970. Original article submitted May 13, 1969.

To solve the instability problem, the energy of plasma fluctuations due to radiation δq_s must be calculated for the case when the equilibrium state can be described by the approximation of radiative thermal conductivity, but the wavelength of the oscillations $\lambda_{min} < l$. Starting from the expression for q_s [4],

$$q_s = \int_0^\infty d\nu \int d\Omega \varkappa_\nu' (I_{\nu p} - I_\nu) \tag{1}$$

we write down an expression for the fluctuations δq_s in the general form

$$\delta q_s = \int_0^\infty d\nu \int d\Omega \{\varkappa_\nu'^\circ (\delta I_{\nu p} - \delta I_\nu) + \delta\varkappa_\nu' (I^\circ_{\nu p} - I_\nu^\circ)\} \tag{2}$$

where the symbol δ denotes variation with respect to density and temperature,

$$\delta \equiv \delta\rho \frac{\partial}{\partial\rho} + \delta T \frac{\partial}{\partial T}$$

For conditions in which the approximation of radiative thermal conductivity is applicable $I_{\nu p}^\circ - I_\nu^\circ \ll I_{\nu p}^\circ$, and the equation of radiative transfer

$$\Omega\nabla I_\nu^\circ = \varkappa_\nu'^\circ (I^\circ_{\nu p} - I_\nu^\circ) \tag{3}$$

has the following solution for the intensity of radiation:

$$I_\nu^\circ = I^\circ_{\nu p} - l_\nu' \Omega\nabla I^\circ_{\nu p} + l^{\nu\prime}\Omega\nabla l^{\nu\prime}\Omega\nabla I^\circ_{\nu p} + \ldots (l_\nu' = 1/\varkappa_\nu')$$

The radiative transfer equation, linearized with respect to perturbations, has the form

$$\Omega\nabla\delta I_\nu = \varkappa_\nu'^\circ (\delta I_{\nu p} - \delta I_\nu) + \delta\varkappa_\nu' \left(\frac{1}{\varkappa_\nu'^\circ}\Omega\nabla I^\circ_{\nu p} - \frac{1}{\varkappa_\nu'}\Omega\nabla\frac{1}{\varkappa_\nu'}\Omega\nabla I^\circ_{\nu p}\right) \tag{4}$$

In the case under consideration, $\lambda < l$ and the term $\varkappa_\nu'^\circ \delta I_\nu$ can be neglected in comparison with c $\Omega\nabla\delta I_\nu$. The quantity $\delta I_{\nu p}$ is proportional to δT; as regards $\delta\varkappa_\nu'$ it contains both temperature fluctuations and density fluctuations. The fact that $I_{\nu p}^\circ - I_\nu^\circ$ is small compared with $I_{\nu p}^\circ$ leads to the inequality

$$\nu'^\circ \frac{\partial I_\nu}{\partial T} \gg \frac{\partial\varkappa_\nu'^\circ}{\partial T} (I^\circ_{\nu p} - I_\nu^\circ) \tag{5}$$

In what follows, it will be shown that

$$\nu'^\circ \frac{\partial I^\circ_{\nu p}}{\partial T} \delta T \gg \frac{\partial\varkappa_\nu'^\circ}{\partial\rho} (I^\circ_{\nu p} - I_\nu^\circ)\delta\rho \tag{6}$$

For the majority of radiative mechanisms and, in particular, for bremsstrahlung and recombination mechanisms, which play a fundamental part in the conditions under consideration, inequality (6) is equivalent to requiring that $\rho\,\delta T \gtrsim T\delta\rho$. Thus to justify (6) we have to show that, for the oscillations being considered, the relative fluctuations of density do not exceed the temperature fluctuations. Assuming that (6) is fulfilled and taking (5) into account, we can simplify (4)

$$\Omega\nabla\delta I_\nu = \varkappa_\nu'^\circ \delta I_{\nu p} \tag{7}$$

Since $\lambda < l_\nu'$ for the majority of quanta, it follows from (7) that $\delta I_\nu \ll \delta I_{\nu p}$ and

$$\delta q_s = \int_0^\infty d\nu \int d\Omega \varkappa_\nu'^\circ \delta I_{\nu p} = \delta T \int_0^\infty d\nu \int d\Omega \varkappa_\nu'^\circ \frac{\partial I_{\nu p}}{\partial T} \tag{8}$$

Thus, for the case in which the wavelength of the oscillations (even in one dimension) is small compared with the free path lengths of the quanta making the fundamental contribution to the radiation, the fluctuations δq_s are determined only by temperature fluctuations for a medium which is optically dense in equilibrium conditions. A concrete calculation of δq_s for bremsstrahlung in a plasma gives

$$\delta q_s = \gamma \frac{Z^3\rho^2}{M^2 T^{1/2}} \delta T, \qquad \gamma \approx 10^{-27} \tag{9}$$

Equation (8) for δq_S shows that when carrying out an analysis for the overheat instability, we can use the dispersion relation (3.9) of paper [1] in the case under consideration, setting

$$\frac{\partial q_{s0}}{\partial \rho_0} \equiv 0, \quad \frac{\partial q_{s0}}{\partial T_0} \equiv \frac{\delta q_s}{\delta T} = \int_0^\infty d\nu \int d\Omega \varkappa_\nu'^\circ \frac{\partial I_{\nu p}}{\partial T} \tag{10}$$

We note that in this case $\delta q_{S0}/\delta T_0$ is no longer a temperature derivative of the equilibrium energy loss value. Only a formal analogy is used in writing down the system of linearized equations. As in the case of a plasma transparent under equilibrium conditions, both high-frequency and low-frequency instabilities occur. The first of these is not associated with the motion of perturbed regions of the plasma, while the second is accompanied by motion of this type. The corresponding frequencies have the form

$$\omega_{1,2} = -\frac{ic^2k^2}{8\pi\sigma_0 t^2} \mp \frac{i}{2}\left[\left(\frac{c^2k^2}{4\pi\sigma_0 t^2}\right)^2 + \frac{2}{3}\frac{c^2k^2}{6\pi\sigma_0 P\alpha t^2}\left(\frac{3}{2}\frac{j^2}{\sigma} - 4\pi T\int_0^\infty d\nu \varkappa_\nu'^\circ \frac{\partial I^\circ_{\nu p}}{\partial T}\right)\right]^{1/2}$$
$$\alpha = \begin{cases} 1, \\ 3, \end{cases} \qquad t^2 = \begin{cases} 1 & (\omega \gg kv_s) \\ 1 + {}^3/_5 v_A^2/v_s^2 & (\omega \ll kv_s) \end{cases} \tag{11}$$

Since, in equilibrium conditions, Joule heating is compensated by radiation close to black-body radiation, we have

$$\frac{j^2}{\sigma} \gg 4\pi T \int_0^\infty d\nu \varkappa_\nu'^\circ \frac{\partial I_{\nu p}}{\partial T}$$

and the instability always occurs.

Finally, we must justify the conclusion drawn above concerning the relation between the fluctuations δT and $\delta\rho$. An appropriate justification can be made most simply for the case of a plane discharge. In the zero-th approximation of geometrical optics, the results are independent of the geometry of the discharge. If the system of linearized equations of magnetohydrodynamics [1] with $m = k_z = 0$ is used, it is not difficult to show that there is a connection between the fluctuations $\delta\rho$ and δT (all terms higher than the zero order of smallness are neglected in geometrical optics):

$$\frac{\rho_1}{\rho_0}\left\{\omega^2\left(\omega^2 + \frac{i\omega c^2k^2}{4\pi\sigma_0} - k^2v_A^2\right) - k^2v_s^2\left(\omega^2 + \frac{i\omega c^2k^2}{4\pi\sigma_0}\right)\right\} = \frac{T_1}{T_0}k^2v_s^2\left(\omega^2 + \frac{i\omega c^2k^2}{4\pi\sigma_0}\right) \tag{12}$$

When $\omega \ll kv_S$ and $\omega \ll kv_S$, the assumption made above can be justified directly from (12).

Thus, it is obvious that short-wavelength overheats with wavelengths $\lambda \ll l$ exist in a discharge which is optically dense under equilibrium conditions. Although these instabilities are less characteristic in an optically dense discharge than in a transparent one, since they can be substantially restricted with respect to the wavelengths, nevertheless, in the conditions under consideration these wavelengths are still not sufficiently small for them to be damped by electron thermal conductivity.

In conclusion, the author wishes to thank A. A. Rukhadze for useful discussions.

LITERATURE CITED

1. V. B. Rozanov, A. A. Rukhadze, and S. A. Triger, "The theory of equilibrium and stability of a heavy current discharge in a dense optically transparent plasma," PMTF, 9, No. 5, 1968.
2. A. A. Rukhadze, and S. A. Triger, "The equilibrium and stability of a heavy current discharge in a dense plasma for conditions of radiative thermal conductivity," PMTF, 9, No. 3, 1970.
3. A. A. Rukhadze and S. A. Triger, "Constrictions in a plasma of finite conductivity," ZhÉTF, 56, no. 3, 1970.
4. Ya. B. Zel'dovich and Yu. P. Raizer, The Physics of Shock Waves in High-Temperature Hydrodynamic Phenomena [in Russian], Nauka, Moscow, 1966.